Models and methods for solving an assignment and vehicle scheduling problem

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Outline

- ✓ Introduction
- ✓ A real assignment and vehicle scheduling problem derived from the operation of a mass transit system (MIO) in Colombia.
- ✓ A MILP formulation of the problem
- ✓ Some scientific challenges

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OR for Development Section

A note from the Developing Countries Committee Chairperson, IFORS VP Sue Merchant

IFORS: Continuing to Pursue Activities in OR for Development

In addition to monitoring progress on ICORD (International Conference on OR for Development) 2014 and the IFORS Prize for OR in Development, the Developing Countries Committee has been busy in the last few months planning a variety of events and supporting others. As you can see in this issue, Hans Ittman from South Africa has been invited and supported by IFORS to give two presentations at a workshop in Senegal.

The DCC Resources website has been updated with new material, thanks to Gerhard Wilhelm Weber for encouraging several of his colleagues and contacts to search out new material and to check the operation of the website; to Prof. Yindong Shen for supplying a copy of her paper on bus transportation and to the publishers Wiley for allowing us to include this until Dec 2014; to our website editor Ruel Tan for adding a section on material recently uploaded to the site, plus a system for rating the papers accessed and a revised counting system for site visits so that these can be monitored more closely.

The committee is also starting to plan for ICORD 2015 and amongst other ideas has been exploring ways of giving developing countries free access to published papers and the possibility of arranging a teachers workshop in the year following the Barcelona conference.

The Vehicle Scheduling Problem (VSP) in Public Transport

Addresses the task of assigning buses to cover a given set of timetabled trips.

Routing problems give special importance to the successive moves of a vehicle, generally referred to displacements in space.

Scheduling problems: a time is associated with each activity, for example, specific times to delivery, in each location.

The sequencing of vehicle activities in both space and time is at the heart of the vehicle scheduling problem

VSP are NP-hard

Why the NP-hard problems are very difficult to solve?

TCF/n	10	20	30	40	50	60
n	.00001	.00002	.00003	.00004	.00005	.00006
n^2	.0001	.0004	.0009	.0016	.0025	.0036
n^3	.001	.008	.027	.064	.125	.216
n^5	.1	3.2	24.3	1.7m	5.2m	13.0m
2^n	.001	1.0	17.9	12.7d	35.7y	366s
3^n	.059	58m	6.5y	3855s	2×10^8s	1.3×10^{13}s

Why the NP-hard problems are very difficult to solve?

TCF	C1	C2	C3
n	**N1**	100N1	**1000N1**
n^2	N2	10N2	31.6N2
n^3	N3	4.64N3	10N3
n^5	N4	2.5N4	3.98N4
2^n	N5	N5+6.64	N5+9.97
3^n	**N6**	N6+4.19	**N6+6.29**

Problem definition

A set of timetabled trips with fixed travel (departure and arrival) times; start and end locations; traveling times between all pairs of end stations.

Objective: to find an assignment of trips to vehicles such that:

- ✓ each trip is covered exactly once.
- ✓ each vehicle performs a feasible sequence of trips.
- ✓ the overall costs are minimized.

An optimal schedule is characterized by minimal fleet size and minimal operational costs

Problem definition

The overall costs can be divided into:

✓fixed costs of vehicles (e.g. investment, maintenance).

✓operational costs (e.g. fuel, attrition, vehicle milage, travel time.

Some definitions used

A trip: each one-way traversal of a route (line). Trips may be characterized by:
start time and start location
end time and end location

A block: One continuous chain of trips that start and end in depots.

A task: A sequence of consecutive travels of a route between two stations: initial and final.

A deadhead trip: travel without passengers (for example, for a depot to a station or station to depot).

Some definitions used

Two types of deadhead trips:

Pull-out trip: Movement of a bus from a depot to the start location of a trip (task)

Pull-in trip. Movement of a bus from an end location of a trip (task) to a depot

Some definitions used

Two types of compatibility relation between two trips (tasks):

$i\alpha j$:—if trip j can be served after trip i, by the same vehicle, considering that:
arrival time of trip i +travel time between end station of i and start station of j $\leq$ departure time of j)

$i\beta j$:—if trip j can be served after trip i, but empty trips for the change of the bus location (deadheading), are not included.

Several extensions for the VSP

✓Multidepot (MDVSP)
✓A heterogeneous fleet with multiple vehicle types
✓Variable departure times of trips
✓More than an operator

✓Different additional requirements.
✓Further restrictions on the routes of the buses.
✓Other objectives.

Examples:
✓In MDVSP the initial and end depot for each trip could be different.
✓Restrictions on maximum travel time of a bus.

The multi depot vehicle scheduling (MDVSP)

The main difference between single and multi depot models is that, in the case of multiple depot, vehicles are housed at different depots.
The objective is to determine the minimum number of vehicles to serve all trips and to identify the optimal locations of the vehicles in order to minimize the total cost.

Two common used approaches:
''Cluster First–Schedule Second'' (Carraresi and Gallo, 1984)
''Schedule First–Cluster Second'' (Gavish and Shifler, 1978).

The multi depot vehicle scheduling (MDVSP)

Formulated as a mixed integer programming problem in two different ways:

‘‘Trip Based’’ formulations, in which the trips are the components to which the variables are related

‘‘Block Based’’ formulations, in which the blocks serve that purpose (least used)

Some formulations and methods for the MDVSP (trip-based formulation)

1. As a multi-commodity formulation. A multi-commodity formulation is a network flow formulation that accounts for multiple commodities shipped from different origins to different destinations in the network. (Mesquita and Paixao, 1990) .

2. As a multi-commodity problem, and a heuristic for solving this problem (Bertossi et al.,1987; Lamatsch, 1990:)

3. Large real world problems using a specific type of column generation called ‘‘Lagrangean Pricing’’ (Lobel, 1997). The model does not take into account route time constraints

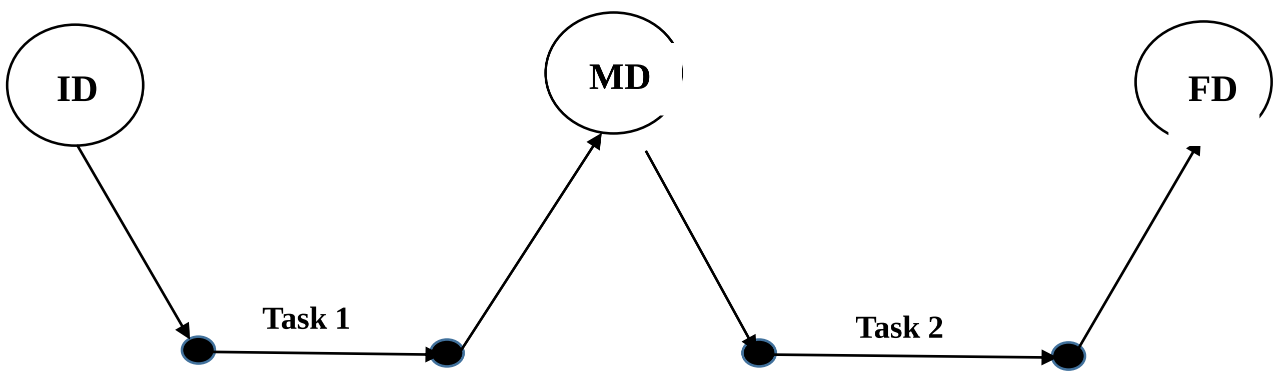
Some formulations and methods for the MDVS (block-based formulation)

1. A formulation with route time constraints (Haghani and Banihashemi, 2002). The authors use a three-step process to solve: reducing the size of the problem (by reducing the number of trips and hence the number of variables), set up the MDVSP problem based on the revised trip data obtained in the first step, and using CPLEX software to solve the problem iteratively by adding appropriate constraints to eliminate violated blocks

Issues not considered in any of the reviewed articles

In approaches for MDVSP models

1.The initial and end depots may be different and the bus should go to a depot when finish each task, not the complete block. Then, the compatibility relation between two tasks, depends also of the middle depot, taking into account the distances between the first task and middle depots in Pull-in trips and the second task and middle depots in Pull-out trips.



Issues not considered in any of the reviewed articles

2. More than one operator, which imposes additional restrictions on:

- ✓ The classic assignment of tasks to buses (of each operator).
- ✓ The classical constraints of depots capacity, adding capacity constraints for buses from other operators in the depot managed for each operator.
- ✓ The classical objective functions

3. In the same model:

- ✓ Different type of tasks
- ✓ Multidepot
- ✓ Multioperator

with new operational restrictions.

An assignment and vehicle scheduling problem derived from the operation of the mass transit system (MIO) in Cali city

Types of buses

Context

There are **four operating companies of the system** with 3 types of buses and four depots (very distant from each other)

Each depot is operated by a company

Two types of tasks should be assigned to operators' buses: complete tasks (CT) or tasks in block (BT)

Tasks in block : A bus serves at most two compatible tasks

All tasks should start and finish in a depot, **not necessarily the same**.

Context

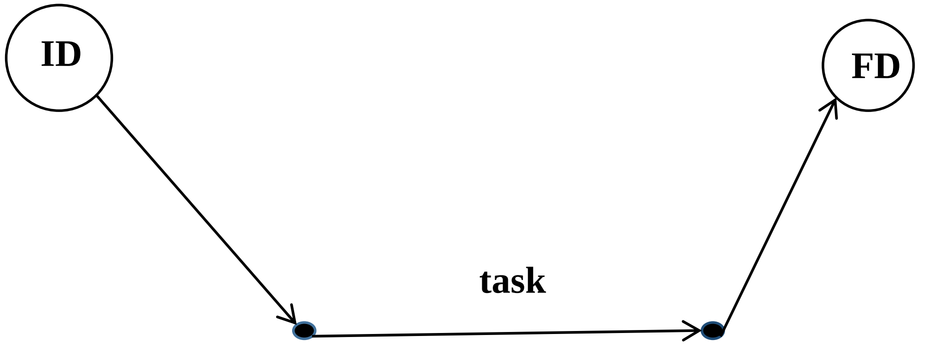
Each type of vehicle can service the two types of tasks

Each vehicle type may be considered heterogeneous because some buses are equipped to carry disabled people and others do not. Some tasks must be performed by buses equipped to transport disabled persons

There are capacity constraints of depots, as well as **a minimum number of parking spaces for buses of each operator in its own depot**.

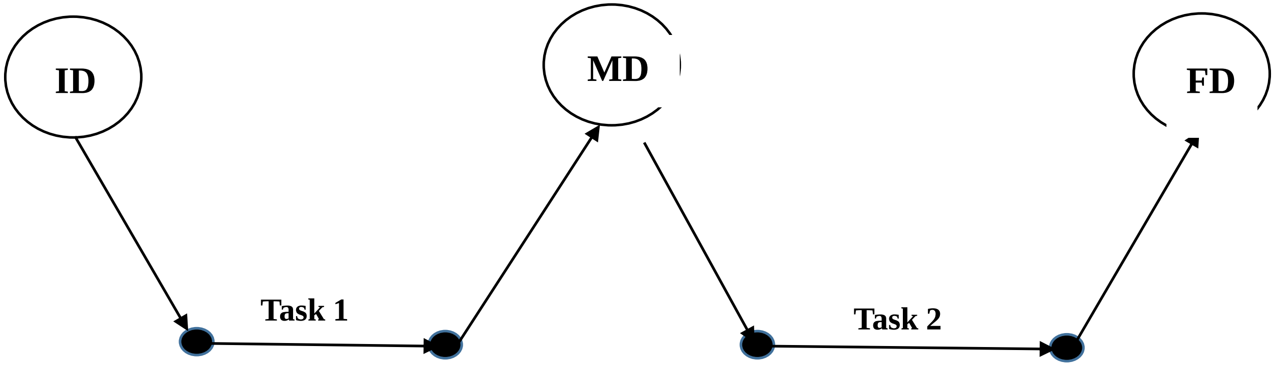
Context

Representation of a "tour" in complete task:



ID: Initial depot
MD: Intermediate depot
FD: Final depot

Representation of a "tour" in a block of tasks:



Context

Two types of travel distances are considered: kilometers associated to tasks (commercial distances) and kilometers associated to distances between depots and stations where tasks should start or end (deadhead distances).

Each operating company has a percentage of participation in the system, in accordance with the number of its buses in the fleet. Then, in accordance with those percentages, the commercial and deadhead distances should be distributed to each operator, when the tasks are assigned to buses.

Context

There are two main objectives defined by the operators

1. Minimize the total deadhead kilometers between depots and stations where tasks should start or end.

2. **Minimize the maximum deviation of Kilometers (commercial and deadhead) assigned to the operators in relation to the ideal they should have, according to their participation with their buses in the fleet**.

The second objective produces a min-max function which needs to be linearized

A MILP formulation of the problem

Restrictions:

✓Each task is assigned exactly to a bus
✓One vehicle can be only assigned to a complete task or to a block
✓A block includes at most two compatible tasks
✓Each task should start and end in some depot, not necessarily the same
✓When a bus finishes the first task of a block, it should go to the same depot from which it should leave to start the second task of that block
✓Each depot has a maximum number of vehicles (capacity) for each operator
✓Special tasks should be assigned to special buses

Some scientific challenges

1. To find stronger formulations

On the theoretical side, the difficulty of a MIP, at least for the branch and cut algorithm uses, can be measured in terms of the weakness of the formulation. This involves the differences between the polyhedron associated with the relaxation of the formulation, and the polyhedron associated with the convex hull of the integer feasible solutions to the MIP. A stronger formulation will typically have a smaller difference between these polyhedrons. And, in some cases, one can provide precise mathematical characterizations of the two polyhedrons.

Some scientific challenges

2. Some aspects for successful algorithm performance

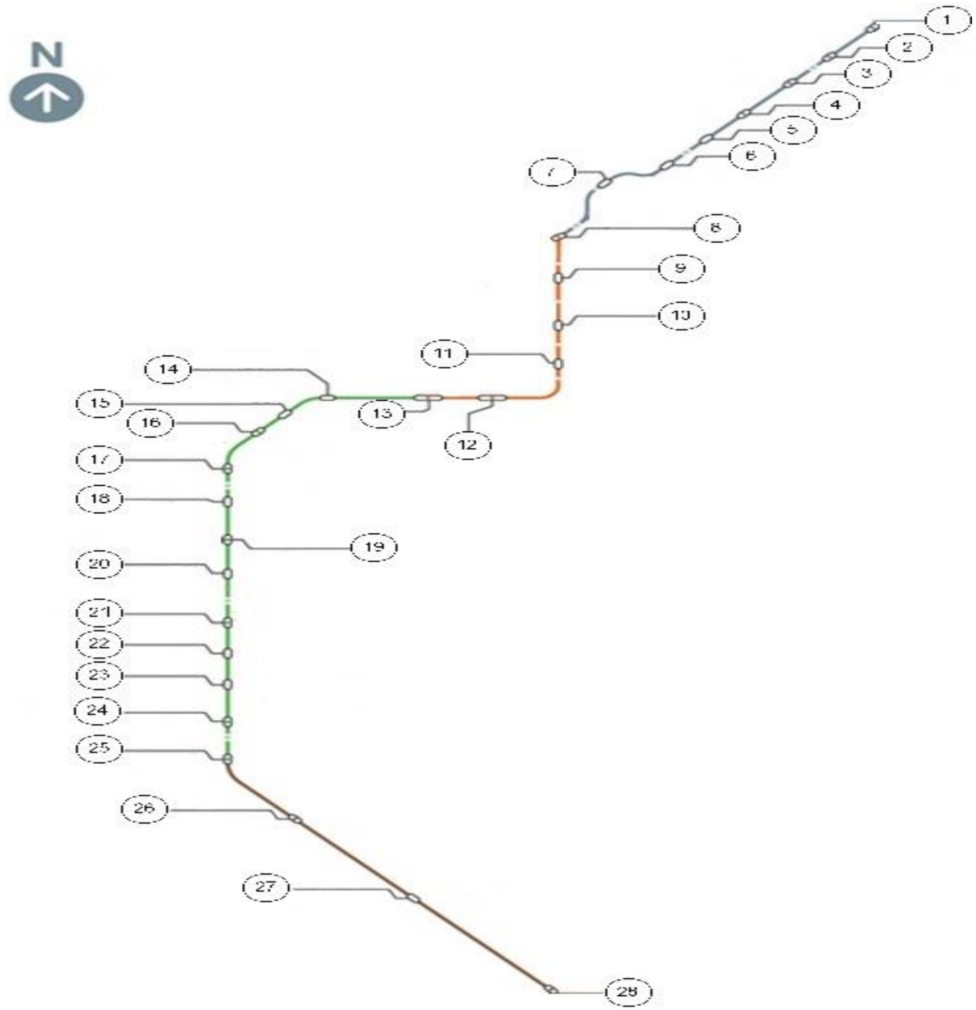
- ✓ Numerical stability and ill conditioning

- ✓ Degeneracy

3. Approaches to solve the multiobjective problem

Real problem: The design of adequate routes and frequencies of buses, in a trunk corridor, of the mass transit system (MIO) in Cali city

A real corridor of the SITM in Cali, Colombia



Research work to continue.

1. Modeling and solution methods to:

✓Location problems of some means of public transport, taking into account the coordination with other modes of transport such as BRT systems.

✓Problems of passenger assignment (in congested transit networks or not), routes design and frequency determination, including the integration of some of these problems.

✓The use of bilevel models, multiobjective approaches

2. Sensitivity analyzes proposals to the above problems NP-hard

Research work to continue.

3. Deepen knowledge about the difficulty of the models obtained in aspects such as:

✓Symmetries of solutions

✓Polyhedral structure

✓Model elements more difficult, as constraints and variables, which can be derived in helpful cuts to improve formulations